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VIRTUAL COACHING CLASSES
ORGANISED BY BOS (ACADEMIC), ICAI

FOUNDATION LEVEL
PAPER 3: BUSINESS MATHEMATICS, LOGICAL REASONING & STATISTICS
LINEAR INEQUALITIES

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Discussion Points

- Linear Inequalities
- Optimisation problems
- Graphical work
- Solution of Study material problems

Chapter objective – pg 3.1

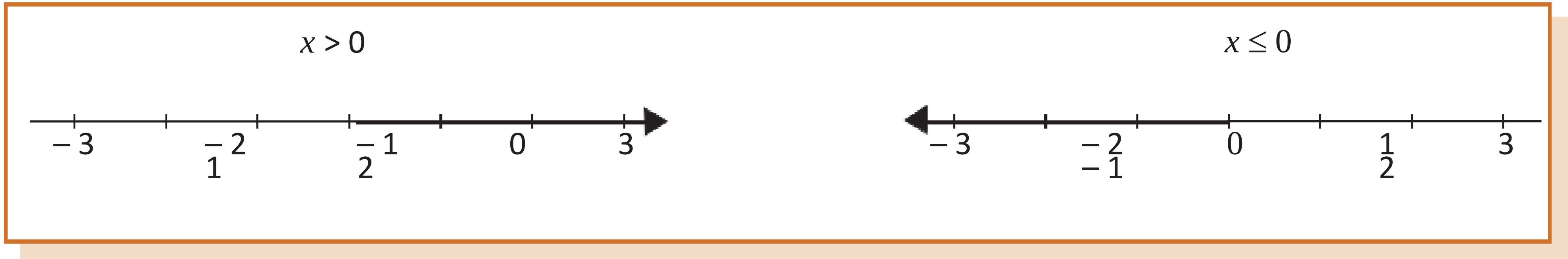
- development of inequations from the descriptive problem;
- graphing of linear inequations; and
- determination of common region satisfying the inequations

INEQUALITIES

- Inequalities are statements **where two quantities are unequal but a relationship exists between them.**
- These type of inequalities occur in business whenever there is a limit on supply, demand, sales etc.
- For example, if a producer requires a certain type of raw material for his factory and there is an upper limit in the availability of that raw material, then any decision which he takes about production should involve this constraint also.
- We will see in this chapter more about such situations.

LINEAR INEQUALITIES IN ONE VARIABLE AND THE SOLUTION SPACE

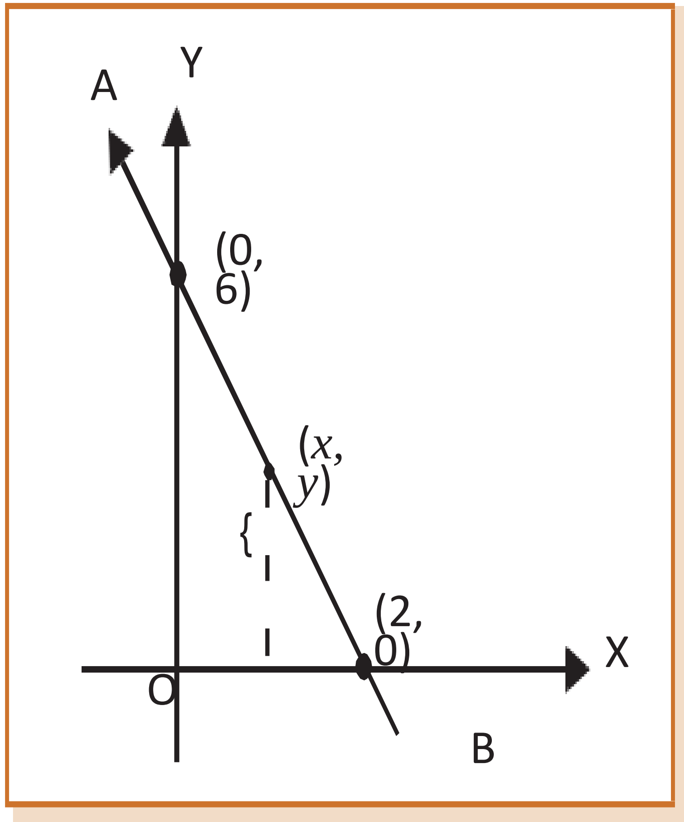
- The values of the variables that satisfy an inequality are called the *solution space*, and is abbreviated as S.S



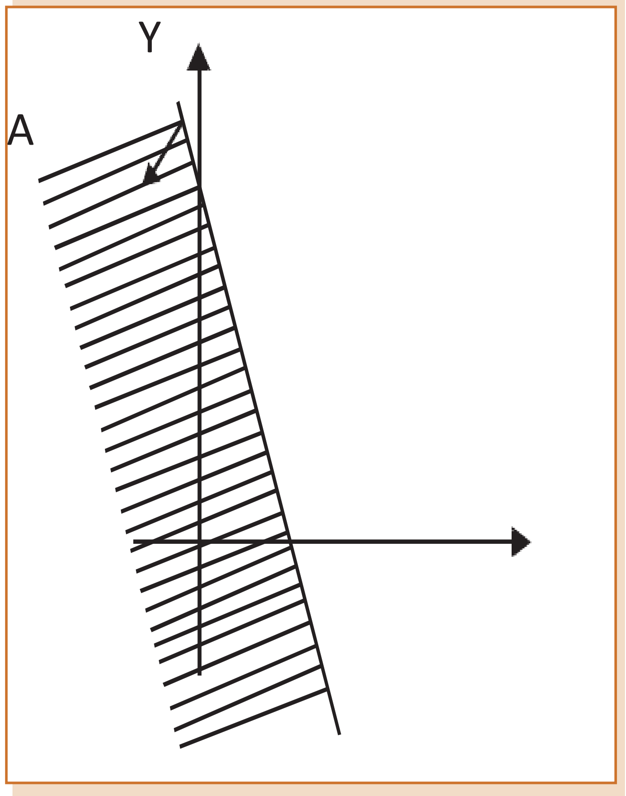
$3x + y < 6$

- inequality in two variables given by $3x + y < 6$
- By trial, we may arbitrarily find such a pair to be (1,1) because 3 ´ 1 + 1 = 4,
- and 4 < 6.

- Linear inequalities in two variables may be solved easily by **extending our knowledge of straight lines.**
- For this purpose, we replace the inequality by an equality and seek the pairs of number that satisfy $3x + y = 6$



- We may write $3x + y = 6$ as $y = 6 - 3x$, and draw the graph of this linear function.
- Let $x = 0$ so that $y = 6$.
- Let $y = 0$, so that $x = 2$.
- Any pair of numbers (x, y) that satisfies the equation $y = 6 - 3x$ falls on the line AB.
- All such points (x, y) for which the ordinate is less than $6 - 3x$ lie below the line AB.



X

O

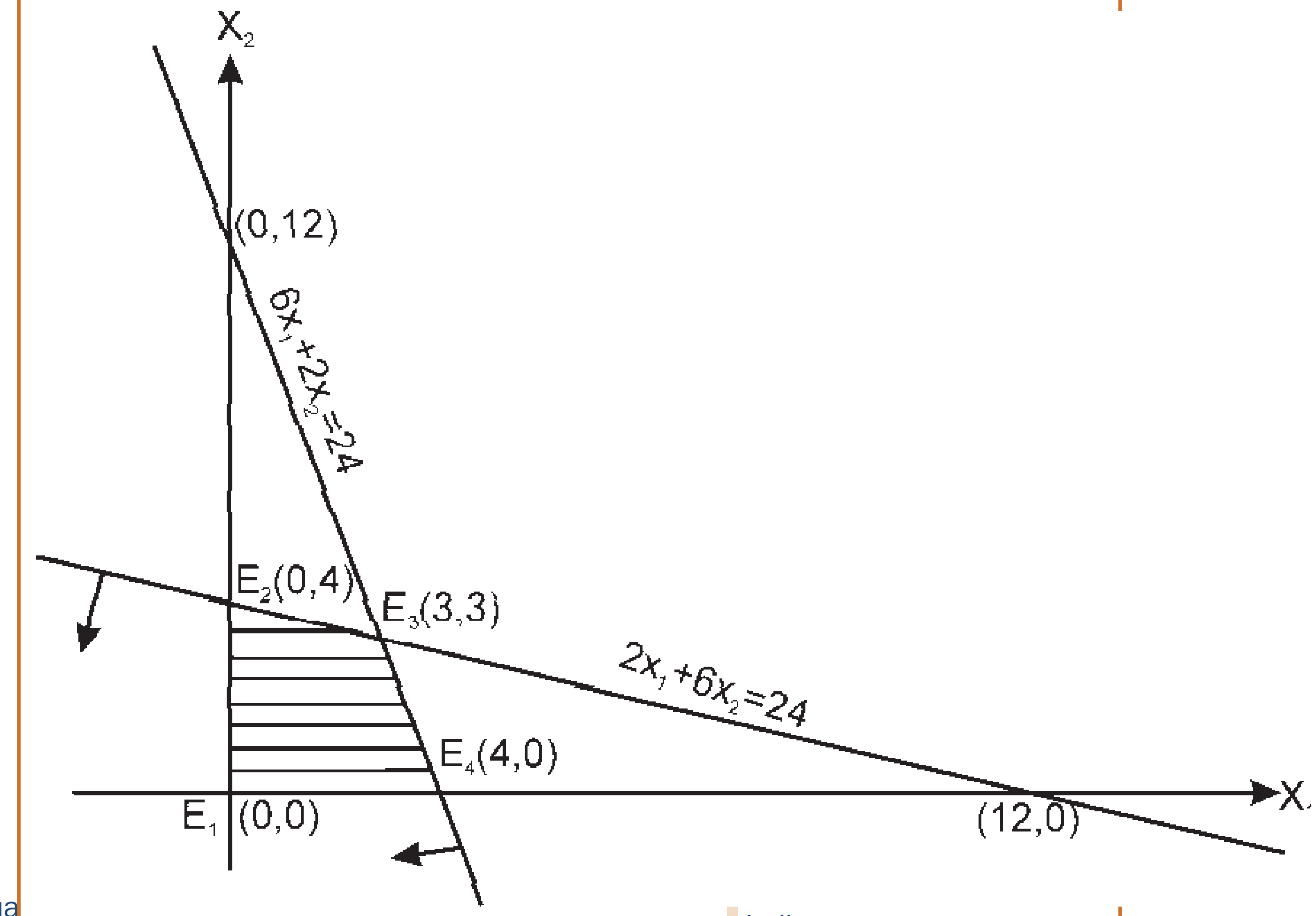
B

Study material pg 3.6 Example

- *A manufacturer produces two products A and B, and has his machines in operation for 24 hours a day. Production of A requires 2 hours of processing in machine M1 and 6 hours in machine M2. Production of B requires 6 hours of processing in machine M1 and 2 hours in machine M2.*
- *The manufacturer earns a profit of ` 5 on each unit of A and ` 2 on each unit of B. How many units of each product should be produced in a day in order to achieve maximum profit?*
- **Solution:**
- Let $x1$ be the number of units of type A product to be produced, and $x2$ is that of type B product to be produced. The formulation of the L.P.P. in this case is as below:
- Maximize $Z = 5x1 + 2x2$
- subject to the constraints. $2x1 + 6x2 < 24$
- $6x1 + 2x2 < 24$
- $x1 \geq 0, x2 \geq 0$

- For the line $2x1 + 6x2 = 24$
- Let $x1 = 0$, so that $x2 = 4$ Let $x2 = 0$, so that $x1 = 12$
- For the line $6x1 + 2x2 = 24$
- Let x1 = 0, so that x2 = 12 Let x2 = 0, so that x1 = 4
- Next page = graph

Feasible region= shaded portion



Example: pg 3.8

- A company produces two products A and B, each of which requires processing in two machines.
- The first machine can be used at most for 60 hours, the second machine can be used at most for 40 hours.
- The product A requires 2 hours on machine one and one hour on machine two.
- The product B requires one hour on machine one and two hours on machine two. Express above situation using linear inequalities.

Approach : inequality problem

Solution: Let the company produce, x number of product A and y number of product B. As each of product A requires 2 hours in machine one and one hour in machine two, x number of product A requires $2x$ hours in machine one and x hours in machine two. Similarly, y number of product B requires y hours in machine one and $2y$ hours in machine two

Expressing inequality

- But machine one can be used for 60 hours and machine two for 40 hours. Hence $2x + y$ cannot exceed 60 and $x + 2y$ cannot exceed 40. In other words,
- $2x + y <= 60$ and $x + 2y <= 40$.
- Thus, the conditions can be expressed using linear inequalities.

Example –pg 3.9

- **Example:** A fertilizer company produces two types of fertilizers called grade I and grade II. Each of these types is processed through two critical chemical plant units. Plant A has maximum of 120 hours available in a week and plant B has maximum of 180 hours available in a week. Manufacturing one bag of grade I fertilizer requires 6 hours in plant A and 4 hours in plant B.
- Manufacturing one bag of grade II fertilizer requires 3 hours in plant A and 10 hours in plant B. Express this using linear inequalities.

- **Solution:** Let us denote by $x1$, the number of bags of fertilizers of grade I and by $x2$, the number of bags of fertilizers of grade II produced in a week. We are given that grade I fertilizer requires 6 hours in plant A and grade II fertilizer requires 3 hours in plant A and plant A has maximum of 120 hours available in a week. Thus $6x1 + 3x2 <= 120$.
- Similarly grade I fertilizer requires 4 hours in plant B and grade II fertilizer requires 10 hours in Plant B and Plant B has maximum of 180 hours available in a week. Hence, we get the inequality $4x1 + 10x2 <= 180$.

Example – pg 3.10

- **Example:** Draw the graph of the solution set of the following inequality and equality:
- $x + 2y = 4$.
- $x - y <= 3$.
- Mark the common region

Solution:

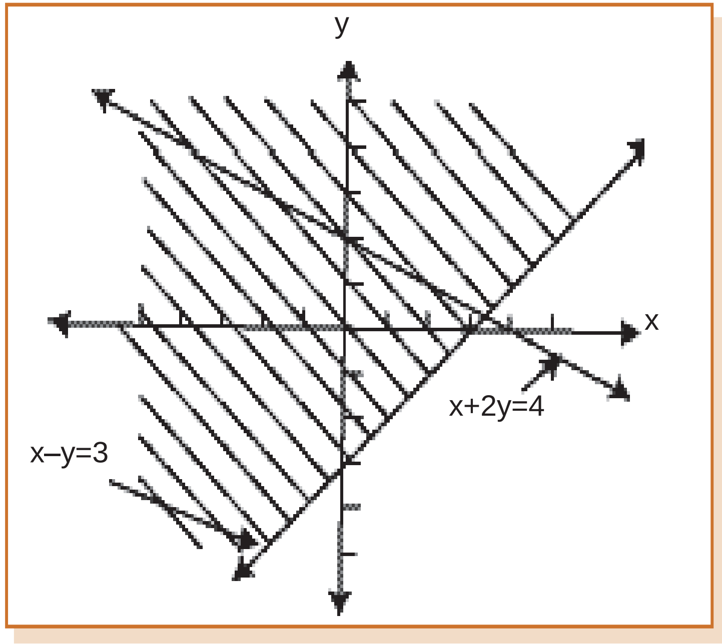
- We draw the graph of both $x + 2y = 4$ and $x - y <= 3$ in the same plane.
- The solution set of system is that portion of the graph of $x + 2y = 4$ that lies within the half- plane representing the inequality $x - y <= 3$.

For x-y=3

x	3	0
y	0	–3

For x+ 2y = 4

x	4	0
y	0	2



Example: pg 3.12

- Two machines (I and II) produce two grades of plywood, grade A and grade B. In one hour of operation machine I produces two units of grade A and one unit of grade B, while machine II, in one hour of operation produces three units of grade A and four units of grade B. The machines are required to meet a production schedule of at least fourteen units of grade A and twelve units of grade B.
- Express this using linear inequalities and draw the graph.

Solution:

- Let the number of hours required on machine I be x and that on machine II be y. Since in one hour, machine I can produce 2 units of grade A and one unit of grade B, in x hours it will produce $2x$ and x units of grade A and B respectively.
- Similarly, machine II, in one hour, can produce 3 units of grade A and 4 units of grade B. Hence, in y hours, it will produce $3y$ and $4y$ units Grade A & B respectively.

Analysis

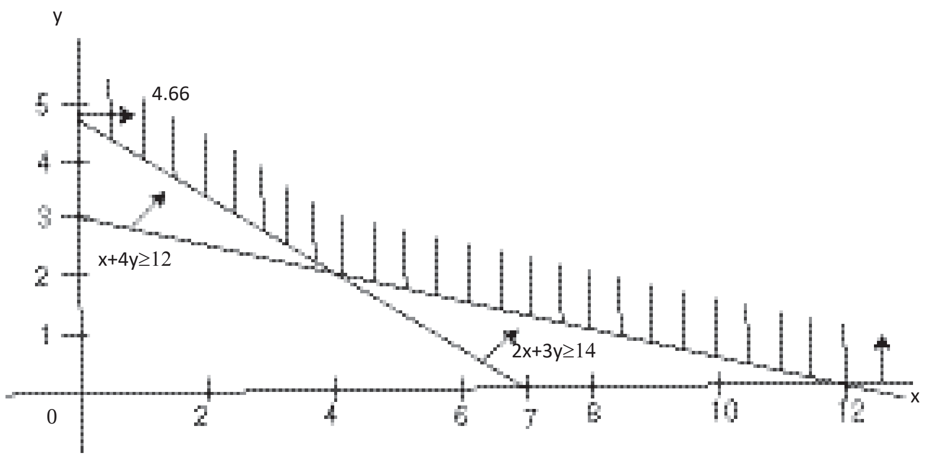
- The given data can be expressed in the form of linear inequalities as follows:
- $2x + 3y >= 14$ (Requirement of grade A)
- $x + 4y >= 12$ (Requirement of grade B)
- Moreover x and y cannot be negative, thus $x > 0$ and $y > 0$

Plotting the graph

- Let us now draw the graphs of above inequalities. Since both *x* and *y* are positive, it is enough to draw the graph only on the positive side.
- For $2x + 3y = 14$, pts are x=7,y =0 and x=0 and y= 4.66
- For $x + 4y = 12$, pts are x=0, y=3 and x=12,y=0

Graph

In the above graph we find that the shaded portion is moving towards infinity on the positive side.
Thus the result of these inequalities is unbounded



THANK YOU